

**Mathematics**  
**Higher level**  
**Paper 1**

Tuesday 12 May 2015 (morning)

Candidate session number

2 hours

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.























Do **not** write solutions on this page.

### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

The function  $f$  is defined by  $f(x) = \frac{3x}{x-2}$ ,  $x \in \mathbb{R}$ ,  $x \neq 2$ .

- (a) Sketch the graph of  $y = f(x)$ , indicating clearly any asymptotes and points of intersection with the  $x$  and  $y$  axes. [4]
- (b) Find an expression for  $f^{-1}(x)$ . [4]
- (c) Find all values of  $x$  for which  $f(x) = f^{-1}(x)$ . [3]
- (d) Solve the inequality  $|f(x)| < \frac{3}{2}$ . [4]
- (e) Solve the inequality  $f(|x|) < \frac{3}{2}$ . [2]

11. [Maximum mark: 16]

Consider the functions  $f(x) = \tan x$ ,  $0 \leq x < \frac{\pi}{2}$  and  $g(x) = \frac{x+1}{x-1}$ ,  $x \in \mathbb{R}$ ,  $x \neq 1$ .

- (a) Find an expression for  $g \circ f(x)$ , stating its domain. [2]
- (b) Hence show that  $g \circ f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$ . [2]
- (c) Let  $y = g \circ f(x)$ , find an exact value for  $\frac{dy}{dx}$  at the point on the graph of  $y = g \circ f(x)$  where  $x = \frac{\pi}{6}$ , expressing your answer in the form  $a + b\sqrt{3}$ ,  $a, b \in \mathbb{Z}$ . [6]
- (d) Show that the area bounded by the graph of  $y = g \circ f(x)$ , the  $x$ -axis and the lines  $x = 0$  and  $x = \frac{\pi}{6}$  is  $\ln(1 + \sqrt{3})$ . [6]



Do **not** write solutions on this page.

**12.** [Maximum mark: 14]

The cubic equation  $x^3 + px^2 + qx + c = 0$ , has roots  $\alpha, \beta, \gamma$ . By expanding  $(x - \alpha)(x - \beta)(x - \gamma)$  show that

(a) (i)  $p = -(\alpha + \beta + \gamma)$ ;

(ii)  $q = \alpha\beta + \beta\gamma + \gamma\alpha$ ;

(iii)  $c = -\alpha\beta\gamma$ .

[3]

It is now given that  $p = -6$  and  $q = 18$  for parts (b) and (c) below.

(b) (i) In the case that the three roots  $\alpha, \beta, \gamma$  form an arithmetic sequence, show that one of the roots is 2.

(ii) Hence determine the value of  $c$ .

[5]

(c) In another case the three roots  $\alpha, \beta, \gamma$  form a geometric sequence. Determine the value of  $c$ .

[6]

**13.** [Maximum mark: 13]

(a) Show that  $\frac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$  where  $n \geq 0, n \in \mathbb{Z}$ .

[2]

(b) Hence show that  $\sqrt{2} - 1 < \frac{1}{\sqrt{2}}$ .

[2]

(c) Prove, by mathematical induction, that  $\sum_{r=1}^{r=n} \frac{1}{\sqrt{r}} > \sqrt{n}$  for  $n \geq 2, n \in \mathbb{Z}$ .

[9]

